Force, Energy, Work, and Power (with U.S. and S.I. units)

**Force**

In physics, force is defined using Newton’s 2nd Law:

\[ F_{\text{net}} = ma \]

where \( m \) is the mass of an object, \( a \) is the object’s acceleration and \( F_{\text{net}} \) is the amount of unbalanced force being applied to the object. The S.I. unit for force is the Newton (N), where

\[ 1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2. \]

Conceptually, 1 N is the amount of force required to move a mass of 1 kg at a rate of acceleration equal to 1 m/s².

Any unit for force should always be made up of a unit for mass (in this case kg) multiplied by a unit for acceleration (m/s²).

In fact, the formal definition of pounds force (lbf, the Imperial unit of force) is

\[ 1 \text{ lbf} = 1 \text{ slug} \cdot \text{ft/s}^2 \]

or the force required to accelerate a mass of 1 slug at a rate of acceleration of 1 ft/s². Since one slug equals 32.174 pounds mass (lb), we get the useful conversion factor:

\[ 1 \text{ lbf} = 32.174 \text{ lb} \cdot \text{ft/s}^2, \]

which is very useful for changing from weights in lbf to masses in lbm, and vice versa.

**Energy and Work**

There are many different types of energy, and different formulas for calculating it, but they should all be measured with the same unit, the Joule.

The following are a few different mathematical definitions of energy.

**Kinetic Energy** = \( \frac{1}{2} mv^2 \)

where \( m \) is mass and \( v \) is speed (or velocity without direction). Kinetic Energy is the energy of motion – anything that is moving has kinetic energy. The higher the speed, the more kinetic energy an object has.

**Gravitational Potential Energy** = \( mgh \)

where \( m \) is mass, \( g \) is gravitational acceleration, and \( h \) is the height of the object relative to a reference point (e.g. the ground). The higher something is above the ground, the more potential it has to fall, increase speed, and thus gain kinetic energy.

In general, all forms of energy have something to do with movement, even including thermal and electrical energy, while potential energy is the potential for movement to happen at some point. For example, electric potential is the potential for an object with charge (e.g. an electron) to move through an electric field. Chemical potential energy can be thought of as the energy held in some substances that can be released through a chemical reaction, such as burning a fuel for heat.
**Work** is defined as the change in the level of energy of a system, so it is also measured in Joules. Mechanical work can be measured by this formula:

\[ W = F \cdot s, \]

where \( F \) is the force exerted and, and \( s \) is the distance over which the force was exerted.

So, if I apply a steady force of 100 N on an object, and the object moves 5 m, then the amount of work I did on the object is:

\[ W = 100 \, N \times 5 \, m \]
\[ = 500 \, N \cdot m \]
\[ = 500 \, J \]

The definition of a Joule can be understood easiest based on the above definition of work:

\[ W = F \cdot s \]

Therefore,

\[ 1 \, J = 1 \, N \cdot m \]
\[ = 1 \, kg \cdot \frac{m}{s^2} \cdot m \]
\[ = 1 \, kg \cdot \frac{m^2}{s^2} \]

As an exercise, use the above definitions of kinetic and gravitational potential energy to show that they are measured in the same units.

In fact, another formula for mechanical work is:

\[ W = p \Delta V \]

Where \( p \) is pressure and \( \Delta V \) is the change in volume. The units still work out to be in Joules as long as pressure is measured in Pascals, and volume in \( m^3 \).

**Power**

The concept of power follows directly from work. Power is simply the rate at which work is done and is measured in Watts (W).

\[ \text{Power} = \frac{\text{Work}}{\text{time}} \]

\[ 1 \, W = \frac{1 \, J}{1 \, s} \]
\[ = \frac{1 \, kg \cdot \frac{m^2}{s^2}}{1 \, s} \]
\[ = 1 \, kg \cdot \frac{m^2}{s^3} \]

So, if a generator supplies 500 J of electricity every second, we can call it a 500 W generator. If a light bulb uses up 60 J of energy every second to brighten up our lives, it is a 60 W bulb.
And Back to Energy!

Another common unit for energy worth mentioning is the **kilowatt-hour** (kWh). This is a unit people often see on their electricity bills. The kilowatt-hour may seem like a unit for power, but it is actually energy.

Think of it this way: 1 kW is 1000 W, which is an amount of power. Power is work (or energy change) divided by time. So now, we are multiplying that unit of power by a different unit of time, the hour. Thus, the kWh is energy divided by time, then multiplied by time, bringing us back to energy.

Just for fun (and to practice unit conversions) let’s convert 1 kWh into Joules.

(Remember that $1 J = 1 kg \cdot \frac{m^2}{s^2}$)

$$1 kW \cdot h \times \left[ \frac{1000W}{1 kW} \right] = 1000 W \cdot h = 1000 kg \cdot \frac{m^2}{s^3} \cdot h$$, from the definition of the Watt above.

$$1000 kg \cdot \frac{m^2}{s^3} \cdot h \times \left[ \frac{3600s}{1h} \right] = 3600000 kg \cdot \frac{m^2}{s^3} \cdot \frac{s}{s^2}$$

$$= 3600000 kg \cdot \frac{m^2}{s^2}$$

Therefore 1 kWh is the same as $3.6 \times 10^6 J$

This might give you an idea why we usually use kWh to measure energy consumption. If we measured it using Joules, the numbers would be huge! Also, when you know the “wattage” of all your appliances, it makes it easy to do a quick calculation in kWh for an hour’s use of those appliances, and then multiply by the electric company’s rate per kWh to find out how much it costs to run your air conditioner.

**Problems**

1) What is the specific volume of a gas that has a weight of 1.5 kg and is contained in a cylindrical container with a volume of 2.0 m$^3$? If this is the answer on earth, where $g = 9.807 \text{ m/s}^2$, what is the gas’s new specific volume if I move the cylinder to Phobos, where $g = 0.0015 \text{ m/s}^2$?

2) You measure the weight of a platinum bar with a mass of exactly 10 kg to be 98.12 N. What is the acceleration due to gravity at your location? If you move to a place where $g = 9.807 \text{ m/s}^2$, what will be the weight of the bar?

3) While on the Jovian moon Ganymede, you use a spring scale to measure the weight of a brick as 1.5 pounds. If you then travel to Io and measure it again with the same scale, what will the weight of the brick be (in pounds)? (Assume the gravitational acceleration is 1.8 m/s$^2$ on Io and 1.4 m/s$^2$ on Ganymede.) Also, given that the brick is a cube with a length of 50 cm on each side, what is its density in kg/m$^3$?
Answers

**Question 1 (weight, mass, density, etc.)**

Given:
- \( v = ? \) (specific volume)
- \( m = 1.5 \text{ kg} \)
- \( V = 2.0 \) (net volume)
- \( g = 9.807 \text{ m/s}^2 \)

Equation:
\[
\text{Specific volume} = \frac{\text{net volume}}{\text{mass}}
\]

\[
v = \frac{V}{m} = \frac{2.0 \text{ m}^3}{1.5 \text{ kg}} = 1.333 \text{ m}^3/\text{kg}
\]

Therefore the specific volume on earth is 1.7 m³/kg. The specific volume on Phobos will be exactly the same as on Earth because specific volume only depends on mass and net volume, which both remain the same regardless of change in location or gravity.

**Question 2 (weight, mass, density, etc.)**

Given:
- \( m = 10 \text{ kg} \)
- \( \text{Weight} = F_g = 98.12 \text{ N} \)
- \( g = ? \)

Equations:
\[
F = ma
\]
\[
F_g = mg
\]

\[
g = \frac{F_g}{m} = \frac{98.12 \text{ N}}{10 \text{ kg}} = 9.812 \text{ m/s}^2
\]

Therefore, the acceleration due to gravity at this location is 9.812 m/s².

Now, we are told that \( g = 9.807 \text{ m/s}^2 \), and need to find the new value for weight \( (F_g) \)

Given:
- \( m = 10 \text{ kg} \)
- \( g = 9.807 \text{ m/s}^2 \)
- \( \text{Weight} = F_g = ? \)

Equations:
\[
F_g = mg
\]

\[
F_g = (10 \text{ kg}) \left( 9.807 \frac{\text{m}}{\text{s}^2} \right) = 98.07 \text{ N}
\]

Therefore, the weight of the platinum rod is now 98.07 N at this new location.
**Question 3 (weight, mass, density, etc.)**

On Ganymede:

Given:

\[ F_g = 1.5 \text{ lbf} \times \left( \frac{32.174 \text{ lbf} \cdot \text{ft}}{1 \text{ lb}} \right) = 48.261 \frac{\text{lbm} \cdot \text{ft}}{s^2} \]

\[ g = 1.4 \frac{m}{s^2} \times \frac{3.28 \text{ ft}}{1 \text{ m}} = 4.592 \frac{\text{ft}}{s^2} \]

\[ m = ? \]

Equations:

\[ F_g = mg \]

\[ m = \frac{F_g}{g} \]

\[ m = \frac{48.261 \text{ lbm} \cdot \text{ft}}{s^2} \times \frac{4.592 \text{ ft}}{s^2} \]

\[ m = 10.5 \text{ lbm} \]

The mass of the brick is 10.5 lbm. We can use this find the weight on Io.

On Io:

Given:

\[ m = 10.5 \text{ lbm} \]

\[ g = 1.4 \frac{m}{s^2} \times \frac{3.28 \text{ ft}}{1 \text{ m}} = 4.592 \frac{\text{ft}}{s^2} \]

\[ F_g = (10.5 \text{ lbm}) \left(4.592 \frac{\text{ft}}{s^2}\right) \]

\[ F_g = 48.2 \frac{\text{lbm} \cdot \text{ft}}{s^2} \times \left( \frac{1 \text{ lb}}{32.174 \text{ lbm} \cdot \text{ft}} \right) \]

\[ F_g = 1.50 \text{ lbf} \]

Therefore, the weight of the brick on Io is 1.50 lbf.

**Density of the brick in kg/m\(^3\):**

Given:

\[ m = 10.5 \text{ lbm} \times \left( \frac{1 \text{ kg}}{2.2 \text{ lbm}} \right) = 4.77 \text{ kg} \]

Length: \( l = 0.5 \text{ m} \)

\[ V = ?, \rho = ? \]

Equations:

\[ \rho = \frac{m}{V} \quad \text{(density = mass/volume)} \]

\[ V = l^3 \]

\[ = (0.5 \text{ m})^3 \]

\[ = 0.125 \text{ m}^3 \]

\[ \rho = \frac{m}{V} \]

\[ \rho = \frac{4.77 \text{ kg}}{0.125 \text{ m}^3} \]

\[ \rho = 38.2 \frac{\text{kg}}{\text{m}^3} \]

Therefore, the density of the brick is 38.2 kg/m\(^3\).