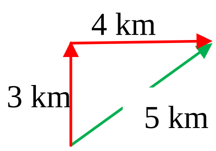
The Physics of Movement (Kinematics)

Rate of Change

When discussing motion there are three terms that help in describing the big picture. These terms are displacement, velocity and acceleration. They all describe in one way or another, a change that is underway or a change that has already happened. All three are also vector quantities as they are associated with a direction.

Displacement

Displacement refers to a straight line distance that an object or body has moved from its point of origin. Displacement does not directly refer to a rate of change but does take into an account a change that has already happened. This is very different than distance as distance is the total movement travelled versus displacement having a magnitude as long as a straight line from the starting point to the end point.



For example if you were to drive 3 kilometers north and 4 kilometers east the total distance you have travelled is 7 kilometers. Your displacement however is only 5 kilometers, as can be seen on the triangle.

The units of displacement are units of length, such as meters (m) or kilometers (km).

Velocity

Velocity refers to the amount of displacement per unit of time an object is undergoing. Velocity can also be described as the rate of change of displacement with respect to time. This means that if you take the derivative of displacement with respect to time you will find your velocity. This shows that the velocity describes how fast you are moving and in what direction.

The units of velocity are a unit of length over a unit of time such as meters per second (m/s) or kilometers per hour (km/h).

Acceleration

Acceleration is the rate of change of velocity with respect to time, or the derivative of velocity with respect to time. The acceleration of an object describes how fast its speed is changing over time and in what direction.

The units of acceleration are a unit of length over a unit of time squared such as meters per second per second (m/s/s or m/s^2).

Kinematics

Kinematics is the study of motion under controlled conditions. There are several formulae that are essential when talking about kinematics and their relationship with displacement, velocity and acceleration. Kinematics deals in uniform motion, where acceleration is usually constant and uniform and the velocity is constant.

Kinematic Formulas

These formulas can be used to solve various kinematic problems. The triangle symbol, 'Δ' known as delta, is a way of saying "the change in".

$\Delta \boldsymbol{x} = (\vec{\boldsymbol{v}}_f + \vec{\boldsymbol{v}}_i)\boldsymbol{t}/\mathbf{2}$	The first equation describes motion when the acceleration is unknown.
$\Delta \boldsymbol{x} = \vec{\boldsymbol{v}}_i \Delta \boldsymbol{t} + \frac{1}{2}\vec{\boldsymbol{a}}(\Delta \boldsymbol{t})^2$	The second formula describes motion where the final velocity of the body in question is unknown.
$\Delta \boldsymbol{x} = \vec{\boldsymbol{v}}_f \Delta \boldsymbol{t} - \frac{1}{2}\vec{\boldsymbol{a}}(\Delta \boldsymbol{t})^2$	The third formula describes motion where the initial velocity of the body in question is unknown.
$\vec{\boldsymbol{v}}_f^2 = \vec{\boldsymbol{v}}_i^2 + 2\vec{\boldsymbol{a}}\Delta \boldsymbol{x}$	The last formula describes motion when the time is unknown.

Linear motion

Using these three terms and the ideas of displacement, velocity and acceleration we can discuss linear motion. Linear motion describes an object moving in no more than one dimension or simply in a straight line. To think about linear motion, think of a car travelling on an open road. This road is completely straight and has no hills. The car may accelerate and decelerate as much as it would like, but it does not change its direction in any way.

For a second example, think of someone walking. This person has displaced himself exactly 100 meters over the past 200 seconds. With this we can solve for the average velocity of the person. We know that average velocity is the total displacement over the total time. Thus we can deduce that the velocity of the sidewalk person is simply 100 meters over 200 seconds. Putting this into the smallest terms we can see that his velocity is 0.5 m/s.

Let us try a more difficult question for which we can use our formulas described above. If a car travelling down a road at a velocity of 20 m/s accelerates at 3 m/s^2 and stops accelerating when he reaches 30 m/s, what distance will it have covered?

We can start by stating the values that we have been given in the problem and then finding which formula we should use in this situation. The givens are:

$$\vec{v}_i = 20\, m/s$$

$$\vec{v}_f = 30\, m/s$$

$$\vec{a} = 3\, m/s^2$$

What we are asked to find is the distance traveled during the acceleration. We need to use the fourth kinematic formula because it contains the three pieces of information we know, as well as the one we are trying to find. (It is also the only formula that does include time, which is not known and we are not required to find it).

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\Delta x$$

$$\vec{v}_f^2 - \vec{v}_i^2 = 2\vec{a}\Delta x$$

$$\frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{a}} = \Delta x$$

By subtracting both sides by the initial velocity and then dividing both sides by two times the acceleration we have isolated for the change in distance and we can now use this new formula to solve for the distance.

$$\frac{30^2 - 20^2}{2(3)} = \Delta x$$

$$\frac{500\, m^2/s^2}{6\, m/s^2} = \Delta x$$

$$83.33\, m = \Delta x$$

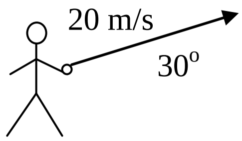
Using the values that were given in the question we can see that during the acceleration the car travelled a distance of 83 ⅓ meters.

Projectile motion

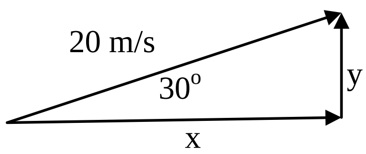
Once linear motion is mastered we can move onto projectile motion. Projectile motion is similar to linear motion but, now there are two dimensions to worry about. Instead of a car driving down a straight road think of a child throwing a baseball in an arc. This baseball now has a velocity in a direction that gives it a component in the 'x' direction and the 'y' direction. This means that you can break up the problem into two linear motion problems, one in the vertical direction and the other in the horizontal direction.

For example, let us evaluate a problem where a child throws a baseball with a speed of 20 m/s in a direction of 30^o off the horizontal. What is the final velocity of the ball when it hits the ground if the child is 1 meter tall?

We can draw a diagram that will allow us to view this problem more effectively and then decide what the best course of action is from there.



First let us look at the initial throw and break it into vectors so that we can analyze each one individually.



Now that we have our right angle triangle, we can find out the x and y magnitudes as below. Look into the forces help sheet to see a more detailed version of vector components.

$$\boldsymbol{x = 20\cos(30)}$$
$$\boldsymbol{x = 17.32\ m/s}$$

$$\boldsymbol{y = 20\sin(30)}$$
$$\boldsymbol{y = 10\ m/s}$$

So the ball is initially traveling 10 m/s upwards and 17.32 m/s to the right. So let us figure out the x direction first because it is usually the easiest in these situations.

Horizontal or x-Direction

Let us talk about the acceleration in the x-direction. The acceleration in this case is equal to zero because there are no external forces acting on the ball after it is released. This means that the horizontal speed of the ball will stay constant from the point of release to the point impact.

Vertical or y-Direction

First off we need to know how high the ball goes in order to find how far it will fall to find its final velocity. Let us discuss the acceleration in the vertical direction. There is only one force pulling on the ball and that is the force of gravity. Therefore the acceleration is 9.8 m/s^2 directly downwards.

We can now use the formulas to find the final velocity in the y direction.

$$\boldsymbol{\frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{a}} = \Delta x}$$

$$\boldsymbol{\frac{-(10^2)}{2(-9.8)} = \Delta x}$$

$$\boldsymbol{\frac{-100\ m^2/s^2}{-19.6\ m/s^2} = \Delta x}$$

$$\boldsymbol{5.1\ m = \Delta x}$$

$$\boldsymbol{6.1\ m = x_{max}}$$

We can use the same formula that was used in our linear motion problem to solve for the maximum height of the ball. At this maximum height, the final velocity will be zero and the acceleration will be -9.8 m/s^2.

Therefore we can see that the ball travels 5.1 m upwards because of the throw, and since the child is 1 m tall, we know that the maximum height of the ball is 6.1 m.

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\Delta x$$

$$\vec{v}_f^2 = 2(9.8\ m/s^2)(6.1\ m)$$

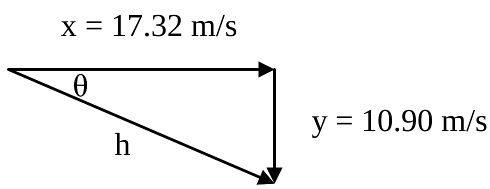
$$\vec{v}_f^2 = 119.56\ m^2/s^2$$

$$\vec{v}_f = 10.9\ m/s$$

Using the value for the maximum height, we can find the final velocity in the vertical direction. In this case we know that at its maximum height, the ball will have an initial velocity of zero and a downward acceleration of 9.8 m/s^2. To find the final velocity of the ball we take the square root of both sides.

Now that we know the final velocity in the y direction we can use what we know from the x direction and determine the velocity as the ball hits the ground.

Resultant Vector



We can use Pythagorean Theorem to solve for the hypotenuse of the triangle to the left and then use trigonometry to find the angle below the x-axis.

$$h^2 = y^2 + x^2$$
$$h^2 = 10.90^2 + 17.32^2$$
$$h^2 = 418.79 m^2/s^2$$
$$h = 20.46\ m/s$$

Using Pythagorean Theorem we find that the magnitude of the resultant vector is 20.46 m/s.

$$\tan(\theta) = \frac{y}{x}$$

$$\theta = \arctan\left(\frac{10.90}{17.32}\right)$$

$$\theta = \arctan\left(\frac{10.90}{17.32}\right)$$

$$\theta = 32.18°$$

Once we have found the magnitude, we can find the direction of the vector using the tangent function. It turns out that the angle is pointing at 32.18^o below the x axis.