

Pounds Mass and Pounds Force

One of the greatest sources of confusion in the Imperial (or U.S. Customary) system of measurement is that both mass and force are measured using the same unit, the pound. To differentiate between the two, we call one type of pound the pound-mass (lbm) and the other the pound-force (lbf). Distinguishing between the two, and knowing how to use them in calculations is very important in using and understanding the Imperial system.

Definition of Mass

The concept of mass is a little difficult to pin down, but basically you can think of the mass of an object as the amount of matter contained within it. In the S.I., mass is measured in kilograms. The kilogram is a fundamental unit of measure that does not come from any other unit of measure.¹

Definition of the Pound-mass

The pound mass (abbreviated as lbm or just lb) is also a fundamental unit within the Imperial system. It is equal to exactly 0.45359237 kilograms *by definition*.

$$1 \text{ lbm} \equiv 0.45359237 \text{ kg}$$

Definition of Force

Force is an action exerted upon an object that causes it to accelerate. In the S.I., force is measured using Newtons. A Newton is defined as the force required to accelerate a 1 kg object at a rate of 1 m/s^2 .

$$1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2$$

Definition of the Pound-force

The pound-force (lbf) is defined a bit differently than the Newton. One pound-force is defined as the force required to accelerate an object with a mass of 1 pound-mass at a rate of 32.174 ft/s^2 . The number 32.174 comes from the acceleration due to gravity on Earth. (Technically, that figure varies depending on where you are standing. “Standard gravity” is defined as exactly 32.174049 ft/s^2 , but usually rounded to 32.174 ft/s^2 .

$$1 \text{ lbf} \equiv 32.174 \text{ lbm} \cdot \text{ft/s}^2$$

What is Weight?

The Weight of an object is the force of gravity currently being exerted on that object. Weight is different from mass because, while the mass of an object is always the same regardless of the object’s location, the weight will be different depending on where it is, and more specifically, how much gravity there is at that location.

So, when you measure weight using pounds, it should be pounds-force that is being measured. However, one of the reasons people use pound-mass and pound-force is that they will be very close to the same number any place on earth, where gravity is always close to 32.174 ft/s^2 .

¹ Originally, the kg was defined as the mass of 100 cm^3 or 1 L of water, and it is still very close to that amount.

Converting between Pounds-mass (lbf) and Pounds-force (lbf)

If you have an object and you are told its mass in lbf, you can find its weight in lbf, provided you know what the acceleration due to gravity is at that location. Technically, this is not a unit conversion since we are changing *what* is being measured (from mass to weight), not just the units.

Here is an outline of the steps to find the weight in lbf, given the mass in lbf and the acceleration due to gravity (g).

1. Use Newton's 2nd Law ($F=ma$) to find weight in $\text{lbf} \cdot \text{ft}/\text{s}^2$.²
2. Convert weight from $\text{lbf} \cdot \text{ft}/\text{s}^2$ to lbf using the relationship $1 \text{ lbf} = 32.174 \text{ lbm} \cdot \text{ft}/\text{s}^2$.

Example 1:

What is the weight of an astronaut with a mass of 180 lbf on the moon, where $g=5.32 \text{ ft/s}^2$?

Step 1: Use Newton's 2nd Law to solve for force of gravity in $\text{lbf} \cdot \text{ft}/\text{s}^2$.

$$\begin{aligned}F &= ma \\F &= 180 \text{ lbf} \times 5.32 \text{ ft/s}^2 \\F &= 957.6 \text{ lbm} \cdot \text{ft}/\text{s}^2\end{aligned}$$

Step 2: Convert $957.6 \text{ lbm} \cdot \text{ft}/\text{s}^2$ to lbf

$$957.6 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2} \times \left| \frac{1 \text{ lbf}}{32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2}} \right|$$

The line above may look a little strange, but if you think of $\text{lbm} \cdot \text{ft}/\text{s}^2$ as a single unit, then you can see how they will cancel out, leaving us with:

$$\begin{aligned}&= \frac{957.6}{32.174} \text{ lbf} \\&= 29.76 \text{ lbf}\end{aligned}$$

Therefore, the astronaut has a weight of 29.76 lbf on the moon.

Example 2:

The astronaut picks up a moon rock and weighs it using a spring scale. The scale says the rock has a weight of 5 lbf.

- a) What is the mass of the rock in lbf?
- b) What is the weight of the rock on Earth in lbf? (use $g=32.174 \text{ ft/s}^2$ for Earth)

For part a, the plan is to find the mass of the rock using the *reverse* of the method used in Example 1:

1. Convert weight from in lbf to $\text{lbf} \cdot \text{ft}/\text{s}^2$
2. Use Newton's 2nd Law ($F=ma$) to find mass in lbf.

² Note that $\text{lbm} \cdot \text{ft}/\text{s}^2$ is a unit for force the same way that $1 \text{ N}=1 \text{ kg} \cdot \text{m}/\text{s}^2$ is a unit for force.

Step 1: Convert weight from 5 lbf to $\text{lbm} \cdot \text{ft}/\text{s}^2$

$$5 \text{ lbf} \times \left| \frac{32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2}}{1 \text{ lbf}} \right|$$

$$= 5 \times 32.174 \left(\frac{\text{lbm} \cdot \text{ft}}{\text{s}^2} \right)$$

$$= 160.87 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2}$$

Step 2: Use Newton's 2nd law to find mass.

$$F = ma$$

$$m = \frac{F}{a}$$

(Remember that the weight of the rock was measured on the moon, so $a=5.32 \text{ ft/s}^2$)

$$m = \frac{160.87 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2}}{5.32 \frac{\text{ft}}{\text{s}^2}}$$

$$m = 30.2 \text{ lbm}$$

Therefore, the mass of the moon rock is 30.2 lbm.

For part b, we use the exactly the same two steps from Example 1 to find the weight of the rock on earth, where $g=32.174 \text{ ft/s}^2$.

Step 1: Use Newton's second law

$$F = ma$$

$$F = 30.2 \text{ lbm} \times 32.174 \text{ ft/s}^2$$

$$F = 973 \text{ lbm} \cdot \text{ft}/\text{s}^2$$

Step 2: Convert 973 $\text{lbm} \cdot \text{ft}/\text{s}^2$ to lbf

$$973 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2} \times \left| \frac{1 \text{ lbf}}{32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2}} \right|$$

$$= \frac{973}{32.174} \text{lbf}$$

$$= 30.2 \text{lbf}$$

Therefore, the weight of the rock on the earth is 30.2 lbf. Why is it the same number as the mass in lbm? Remember, the main reason people use these units is that mass in lbm and weight in lbf will be the same number as long as the object is on Earth (or any place where $g=32.174 \text{ ft/s}^2$).